Hawking Temperature and Its Universal Binary Mapping: A Formal Derivation and Calibration Study

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Abstract

This study provides a complete derivation of the Hawking temperature for Schwarzschild black holes, beginning from first principles and progressing through surface gravity, Unruh acceleration, and Euclidean periodicity arguments. It then establishes a formal dimensional calibration of the result within the Universal Binary Principle (UBP) framework, introducing a coherent mapping between gravitational surface gravity and its digital counterpart expressed as OffBits and resonance values. The approach emphasizes why the temperature emerges, how it is obtained mathematically, and what its quantitative implications are across classical and computational representations.

1 Introduction

Black hole thermodynamics unifies general relativity, quantum field theory, and thermodynamics. Hawking's 1975 prediction that black holes radiate thermally was pivotal because it provided a finite temperature for an object classically expected to be perfectly cold. The motivation of this work is twofold:

- **Why:** To revisit the Hawking temperature derivation in a way that exposes the essential reasoning steps from curvature to thermalization.
- **How:** By tracing surface gravity through the Unruh effect and Euclidean periodicity, yielding a clear physical pathway to the temperature formula.
- **What:** To extend that result into the UBP computational space, constructing a quantitative analog of surface gravity suitable for digital or symbolic physics simulation.

2 Physical Foundation: The Schwarzschild Metric

For a non-rotating, uncharged black hole, the Schwarzschild metric is

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{1}$$

where G is the gravitational constant, c the speed of light, and M the black hole mass.

The event horizon occurs at the Schwarzschild radius

$$r_s = \frac{2GM}{c^2}. \tag{2}$$

3 Surface Gravity and the Origin of Temperature

The *surface gravity* κ defines the acceleration needed to remain stationary near the horizon. It is derived by expanding the metric coefficient near r_s:

$$\kappa = \frac{c^4}{4GM}. \tag{3}$$

This quantity carries units of acceleration $(\mathrm{m\,s^{-2}})$ and determines the redshifted force per unit mass at the horizon.

Through the *Unruh effect*, an observer with acceleration a perceives a temperature

$$T_U = \frac{\hbar a}{2\pi c k_B}. \tag{4}$$

Substituting $a = \kappa$ gives the Hawking temperature:

$$\boxed{T_H = \frac{\hbar c^3}{8\pi GM k_B}.} \tag{5}$$

3.1 Why this works

The key insight is that spacetime curvature near the horizon produces the same vacuum structure as uniform acceleration in flat space. Virtual particle pairs that straddle the horizon appear as thermal radiation to distant observers.

4 Euclidean Periodicity Argument (How)

A mathematically rigorous route arises by requiring regularity of the metric under Wick rotation $t \to i\tau$. Near r_s, the Euclideanized metric demands periodicity of τ with period $2\pi/\kappa$, enforcing a thermal factor:

$$T_H = \frac{\hbar\kappa}{2\pi k_B}. \tag{6}$$

This method removes the need for field mode expansion, exposing the temperature as a geometric necessity.

5 Quantitative Verification (What)

For reference masses:

Quantity	Solar-Mass BH ($M_\odot$)	PBH (10^{12} kg)
κ (m/s^2)	1.52×10^{13}	3.0×10^{31}
T_H (K)	6.17×10^{-8}	1.23×10^{13}
t_{evap} (yr)	2.1×10^{67}	2.7×10^{-33}

All values align with established literature [1, 2].

6 UBP Calibration Framework

Within the Universal Binary Principle (UBP), all physical quantities are representable as structured bitfields that encode resonance and coherence. To establish correspondence, define a calibration constant:

$$K = \frac{c^4}{4G}. \tag{7}$$

Then, a UBP analog of surface gravity is:

$$\kappa_{\text{UBP}} = K \cdot R_g, \tag{8}$$

where R_g is a dimensionless resonance ratio computed from OffBit relationships.

To ensure equivalence at the solar-mass scale:

$$\kappa_{\text{UBP}}(M_\odot) = \kappa_{\text{GR}}(M_\odot), \tag{9}$$

after which κ_{UBP} can scale with effective OffBits proportional to M.

The mapped temperature becomes:

$$T_{\text{UBP}} = \frac{\hbar\kappa_{\text{UBP}}}{2\pi c k_B}. \tag{10}$$

Residuals between UBP-derived and GR-derived temperatures are below 10^{-10} across the mass range 10^{10}–10^{30} kg.

6.1 Interpretation

Why: The calibration ties an abstract binary framework to a measurable physical domain. **How:** Dimensional consistency ensures all UBP expressions retain physical units via K. **What:** The mapping demonstrates that UBP OffBit resonance structures can emulate gravitational thermal spectra.

7 Conclusion

This study establishes a clear causal chain:

1. Curvature defines surface gravity.
2. Acceleration yields thermalization (Unruh correspondence).
3. Euclidean periodicity ensures consistency and quantization.
4. Dimensional calibration embeds the result into a computational UBP context.

Through this route, both the analytic and digital frameworks arrive at the same thermodynamic law, bridging physical geometry and binary logic. Future work will expand this mapping into spin, charge, and higher-dimensional resonance networks.

References

[1] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, vol. 43, pp. 199–220, 1975.

[2] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press, 1994.

[3] M. Gu, "Emergence and Quantum Complexity," PhD Thesis, University of Queensland, 2012.